Figures of Merit of a Locked Tunable Oscillator

Andrey Pluteshko
RF Design
Advantex LLC
Moscow, Russia
drunas@advantex.ru

***Abstract*—The paper discusses quantities to characterize close-in phase noise performance of a locked tunable oscillator in low phase noise applications. It is shown that low phase noise of the oscillator is an inappropriate criterion for the oscillator selection. Correct target quantities are derived and discussed.**

***Index Terms*—PLL, FLL, low noise sources**

I. Introduction

Many low noise applications require a control loop to lock the frequency of a tunable oscillator to a more stable standard. This oscillator is usually a voltage controlled oscillator (VCO). Although the discussion given in the paper is valid for any kind of tunable oscillator, here it is referred to as a VCO, since it is used in the vast majority of loop designs.

In most cases, VCO impairs the noise performance only at the offsets that are not under the loop control. This is due to its selection according to *the lowest integral PM noise criterion*. According to it, once the structure of the loop and its components are defined the last step in the design is the selection of the *free running* VCO with the lowest noise level.

This approach is generally extended to cover ultra low noise applications. For example, the most common VCO in modern atomic fountain clock synthesizers is a dielectric resonator oscillator (DRO).

However, for some ultra low noise applications the locked VCO noise performance at close-in offsets may be of high importance. In [1] a synthesizer is described where the synthesizer's residual Allan deviation (ADEV) of 10^{-15} is required at 1...1000 s averaging times. The synthesizer employs phase locked loop (PLL) to stabilize the DRO frequency. It is said that the DRO (which is generally considered a low noise device suitable for the synthesis in question) was carefully selected to achieve the objective, since very few DROs are up to the task. Therefore, the locked DRO performance at very low close-in offsets was crucial.

In [2] for some offset range one of the best phase noise performances up to date was achieved with the frequency locked loop (FLL). It was implemented to lock the YIG tuned oscillator (YTO). However, the phase noise of the latter put the performance of the whole system under considerable risk. To suppress it, *two* integrators were embedded into the loop. But it is to be remembered that YTO is considered a relatively low noise device, since it is based on the high-Q resonator.

Therefore, when loops with the ultimate noise performance are of concern, the most crucial parameter is the residual close-in PM noise of the *locked* VCO. In this case, the free-running stability performance is misguiding, and the locked VCO performance should be defined by some other quantities.

II. Derivation

A. Loop configuration

To arrive at these quantities, conditions for comparison between VCOs should be stated. Let us assume that each VCO is implemented into a loop containing a detector and an amplifier described by the transfer functions $G_D(s)$ and $G_A(s)$ respectively. (Fig. 1). When the VCO is substituted, the former is not changed, and the latter is adjusted according to some criterion.

The amplifier is assumed to be a non-ideal integrator, realized with an operational amplifier of the finite DC gain A_{DC}. Hence:

$$G_A(s) = A_0 \frac{s + 2\pi f_I}{s + 2\pi f_{DC}}, \tag{1}$$

where A_0 is the broadband gain, f_I is the integrator bandwidth, $f_{DC} = f_I \frac{A_0}{A_{DC}}$. Generally, A_0 and f_I are uniquely correspond to a particular VCO, whereas A_{DC} is defined by the chosen operational amplifier, and, therefore, invariant under the VCO substitution.

Under aforementioned conditions, the locked VCO PM noise $\varphi(t)$ may be due to free-running VCO PM noise $n(t)$ or due to slow drifting fluctuations $d(t)$ imposed on the VCO externally (e.g. temperature or supply voltage drifts *independent* of the VCO selection). In the latter case, a VCO is characterized by its sensitivity to the external drift K_{dr}.

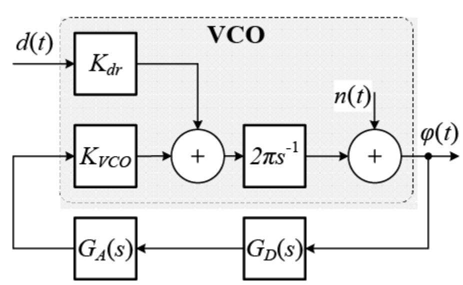

Fig. 1. Linearized model of the control loop containing an amplifier ($G_A(s)$), a detector ($G_D(s)$), and VCO (VCO parameters: K_{VCO} is the tuning sensitivity, K_{dr} is the sensitivity to the external drift $d(t)$, and $n(t)$ is the PM noise).

B. Suppression of fluctuations

It is to be noted that a noisy VCO gives an advantage over the high performance counterpart only if it provides some increase of free-running instabilities *suppression*. Here the open loop transfer function $K(s) = K_{VCO}\frac{2\pi}{s}G_A(s)G_D(s)$ is crucial. Considerable suppression takes place when $|1+K(s)| \gg 1$. Under this approximation, any perturbation is suppressed by factor $|K(s)|$. This permits to write the loop response $\Phi(s)$ to the drift input $D(s)$ as

$$\Phi(s) \approx \frac{D(s)}{G_D(s)}\left(\frac{K_{dr}}{G_A(s)K_{VCO}}\right), \quad (2)$$

and to the noise input $N(s)$ as

$$\Phi(s) \approx \frac{s}{2\pi G_D(s)}\left(\frac{N(s)}{G_A(s)K_{VCO}}\right). \quad (3)$$

Both (2) and (3) have a factor in brackets that contains terms dependent on VCO substitution. This factor has a form of ratio. The numerator describes the influence of the *free-running* VCO, and the denominator the VCO dependent part of the loop gain, $G_A(s)K_{VCO}$. The log frequency response of the latter, $\sigma(f) = 20\log|G_A(j2\pi f)K_{VCO}|$, is discussed in detail below.

C. Comparison criteria

The next step is to consider strategies to configure the loop. Firstly, there may be a requirement not to change significantly $K(s)$. Since A_{DC} is by several orders of magnitude larger than A_0, we can assume $A_{DC} \to \infty$, and, consequently, $G_A(s) \approx A_0\left(1+\frac{2\pi f_I}{s}\right)$ for sufficiently large offset frequencies. In other words, at these offsets $G_A(s)$ given by (1) is numerically indistinguishable from the ideal integrator transfer function.

The second strategy is to maximize the noise suppression. Therefore, $G(s)$ is increased until some critical values are not approached. These are evidently tied to the loop stability. The latter may be affected by VCO inertial properties. Consequently, in this case $G(s)$ should be modified in such a way that amplitude and phase stability margins are left unchanged.

After the qualitative exposition of two strategies, it is reasonable to turn our attention to the quantitative analysis. Let us take two VCOs with K_{VCO1}, K_{VCO2}, and 3 dB modulation bandwidths MBW_1, MBW_2 respectively. Then we introduce two quantities $\kappa = 20\log\frac{K_{VCO2}}{K_{VCO1}}$, and $\mu = 20\log\frac{MBW_2}{MBW_1}$. Moreover, we assume that a noisy VCO is substituted for the low noise one. Consequently, $K_{VCO1} < K_{VCO2}$, and, $MBW_1 < MBW_2$, reflecting the fact that noisy VCOs utilize lower Q-factor resonators.

Let us denote the loop configuration with the low noise VCO as an a configuration. Now, we can enforce two criteria to compare loop performances under the VCO substitution:

1) fixed f_I (b configuration): f_I and A_0K_{VCO} are to be kept constant (curve $\Delta\sigma_{a\to b}$ in Fig. 2 with possible suppression *improvement* of κ at the "drift" offsets, i.e. very low close-in offsets);

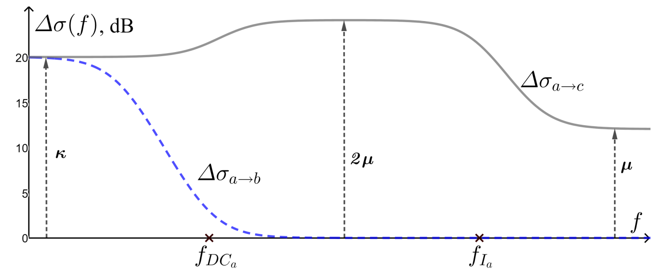

Fig. 2. Differences between instability suppression for fixed ($\Delta\sigma_{a\to b}$) and adjustable ($\Delta\sigma_{a\to c}$) loop bandwidths ($\kappa = 20$ dB, and $\mu = 12$ dB).

2) fixed stability margins (c configuration): f_I and A_0K_{VCO} are scaled by $\frac{MBW_2}{MBW_1}$ to fulfill $K(j2\pi f_{I_a}) = K(j2\pi f_{I_c})$ (curve $\Delta\sigma_{a\to c}$ in Fig. 2 with possible suppression improvement of 2μ). Here we assume the flat response of the detector for $f > f_{I_a}$. (Fig. 2)

D. Figures of merit

The results given the previous subsection make it possible to introduce VCO figures of merit (FOM). They are defined in such a way that a *smaller* FOM indicates a VCO with the *better* locked performance. These are:

1) κ-FOM (to compare the ultimate VCO performances under Criterion 1):
   - for drifts, it is the sensitivity to drift K_{dr} divided by K_{VCO}.
   - for noise perturbation described by its PSD $\mathcal{L}(f)$, it can be defined up to some constant C_κ (same for all VCOs under comparison):

$$\kappa_{FOM}(f) = \mathcal{L}(f) - 20\log(K_{VCO}) + C_\kappa. \quad (4)$$

2) μ-FOM (to compare the ultimate VCO performances under Criterion 2) :

$$\mu_{FOM}(f) = \mathcal{L}(f) - 40\log(MBW) + C_\mu, \quad (5)$$

where C_μ is some constant, analogous to C_κ.

In a sense, the *noise* FOMs are analogous to FOMs that specify the PLL IC noise performance. However, the latter have very uncommon values, say −230 dBc/Hz. This drawback may have been inherited by the *noise* VCO FOMs as well, have the constants C_κ and C_μ. Also, *all* VCO FOMs exhibit a rather interesting property, namely, they are invariant under noiseless frequency multiplication and division. Therefore, they may be used to compare VCOs performances regardless of the actual values of their output frequencies.

In other words, before VCO performances compared all stochastic quantities should *divided* by respective K_{VCO} values. The sensitivities to drift of the external parameters should be normalized by K_{VCO}. For a sensitivity to supply voltage drift K_{push}, the corresponding locked VCO FOM is $\kappa_{push} = K_{push}/K_{VCO}$. For a frequency temperature sensitivity K_{temp}, it is $\kappa_{temp} = K_{temp}/K_{VCO}$. The locked VCO phase noise performances in the close-in range should be compared with regard to the $\kappa_{FOM}(f) = \mathcal{L}(f) - 20\log(K_{VCO}) + C_\kappa$. The difference between VCO noise κ-FOMs $\Delta\kappa_{FOM}(f) = \kappa_{FOM2}(f) - \kappa_{FOM1}(f)$ shows the performance degradation in dB, if VCO2 substituted for VCO1.

To illustrate this principle of VCO comparison, data on four VCOs were collected. These four are a surface acoustic wave (SAW) resonator VCO HFSO1000-5H and DRO SDRO800-8 by Synergy Microwave [3], ceramic resonator VCO (CRO) CRO3060A by Z-Communications [4], and the MMIC HMC505 [5]. The VCO data are in Table I.

Based on them, different κ-FOMs were obtained (Table II). It can be easily seen that VCOs traditionally regarded as stable and low noise (SAW and DRO) cannot compete with VCOs that are usually not considered suitable for ultra low noise applications.

The analogous line of thought is valid for μ-FOMs. Unfortunately, the available data necessary for calculation in this case do not cover sufficiently many types of VCO, and are not given here. However, it should be noted that there are significant indications of advantage of noisy technologies over the low noise ones in this case as well.

TABLE I
DATA ON FREE-RUNNING VOLTAGE CONTROLLED OSCILLATORS (TAKEN FROM DATASHEETS EXCEPT NOTED)

Techn.	**Phase Noise at 1 kHz offset, dBc/Hz**	**VCO Parameters**		
		K_{VCO}, ***MHz/V***	K_{push}, ***MHz/V***	K_{temp}, ***MHz/K***
SAW	–116	0.02	0.02	—
DRO	–80	1.5	0.4	0.04
CRO	–81	14	1	—
MMIC	–46[a]	220[a]	20	0.8

[a]Measured.

TABLE II
DATA ON VOLTAGE CONTROLLED OSCILLATORS FIGURES OF MERIT (NOISE κ-FOMS ARE COMPARED TO CRO PERFORMANCE)

	Noise κ-FOM differences (dB) at		**Drift κ-FOMs**	
Techn.	1kHz	1MHz	κ_{push}	κ_{temp}, ***V/K***
SAW	22	46	1	—
DRO	20	16	0.27	0.027
CRO	0	—	0.07	—
MMIC	11	9	0.09	0.004

III. CONCLUSION

When the locked VCO performance is crucial, a VCO should be chosen according to the specific figures of merit. After the latter having been compared for different VCO types, it turns out that largely available, and usually regarded as noisy, VCOs show considerable advantage over the conventional low noise ones. Further step in the development of the aforementioned ideas may be a theoretical examination of the oscillator circuit parameters that provide the minimal FOM value for a given technology.

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